Modal Logic and the Vietoris functor

Yde Venema

In this tutorial we explain the intimate connections, going back to work of Leo Esakia in duality theory, between modal logic and the Vietoris construction in topology. We start with the most straightforward case, explaining in detail how the basic axioms of modal logic induce a functor on the category of Boolean algebras which is dual to the Vietoris functor on the category of Stone spaces.

In the second part of the tutorial we discuss some variations and generalizations of this basic picture. In particular, we discuss the situation that arises when we change the base categories of Boolean algebras and Stone spaces to certain categories of frames and compact Hausdorff spaces. Finally we show how insights from coalgebraic modal logic, and in particular the concept of relation lifting, can be used to place these results in a more general framework, which is parametrized by a duality of base categories and a functor on the category of sets satisfying certain properties.