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ABSTRACTS BOOK

Plenary Talks

Real Analysis Methods in Ergodic Theory

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On σ-finite measure space $(X, \mathbb{S}, \mu)$, we consider point transformations $T : X \to X$ which are measure preserving: $A \in \mathbb{S} \Longrightarrow T^{-1}(A) \in \mathbb{S}$ and $\mu\big(T^{-1}(A)\big) = \mu(A)$; and ergodic: $\mu\left(T^{-1}(A)\triangle A\right) = 0 \Longrightarrow \mu(A) = 0$ or $\mu(X\backslash A) = 0$, or equivalently $f \circ T = f$ a.e. $\Longrightarrow f =$ Const. During our talk, we briefly describe the main results contained in the monograph [1]. Namely, we present a simple proof of the Individual Ergodic Theorem:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x) = \frac{1}{\mu(X)} \int_X f\, d\mu \qquad \text{for a.a. } x \in X,$$

based on Maximal Ergodic Theorem:

$$\mu\{f^* > \lambda\} \leq \frac{1}{\lambda} \int_{\{f^*>\lambda\}} f\, d\mu \qquad \text{for } \lambda > \frac{1}{\mu(X)} \int_X f\, d\mu,$$

where f^* is the ergodic maximal function:

$$f^*(x) = \sup_{N\geq 1} \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x).$$

The latter theorem will be proved by the "filling scheme" method. The continuous version of this method will be demonstrated by proving the Balanced Ergodic Equality:

$$\mu\{f^* > \lambda\} = \frac{1}{\lambda} \int_{\{f^*>\lambda\}} f\, d\mu \qquad \text{for } \lambda > \frac{1}{\mu(X)} \int_X f\, d\mu,$$

where the ergodic maximal function is defined in the continuous case (for a group of measure-preserving transformations $\{T_t\}_{t\in\mathbb{R}}$) as

$$f^*(x) = \sup_{t>0} \frac{1}{t} \int_0^t f(T_\tau x)\, d\tau.$$

We expose some characteristic features of the ergodic maximal function and ergodic Hilbert transform:

$$Hf(x) = \sum_{n=-\infty}^{\infty}{}' \frac{f(T^n x)}{n}.$$

To this end, a unified approach is developed to prove number of properties of classical Hardy-Littlwood maximal function and Hilbert transform in the ergodic setting.

An elementary proof of the recurrence property of ergodic random walks $S_N f(x) = \sum_{n=0}^{N-1} f(T^n x)$, which is the generalization of classical theorem in Probability Theory, will be demonstrated.

References:

[1] L. Ephremidze: *Real Analysis Methods in Ergodic Theory*, Nova Science Publishers, New York, 2012.

On the Essential Unboundedness in Measure of Sequences of Superlinear Operators in Classes $L\phi(L)$

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We establish a general theorem for a wide class of sequences of superlinear operators about existence of a function g from a certain class $L\phi(L)$ such that the sequence of functions $T_n(g), n = 1, 2, \ldots$ is essentially unbounded in measure on I^2. This theorem implies several results about divergence of sequences of classical operators.

Boundary Values of Functions of Dirichlet Spaces L^1_2 on Capacitary Boundaries

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We adapt a concept of capacitary boundaries (introduced by Gol'dshtein and Vodop'yanov for a study of the boundary behavior of quasiconformal homeomorphisms) to a study of boundary values of Sobolev functions with square integrable weak gradient. We prove that any function of the space $L^1_2(\Omega)$ can be extended quasi-continuously (in the sense of the conformal capacity) to the capacitary boundary of any simply (finally) connected plane domain $\Omega \neq \mathbb{R}^2$. We use the Riemann Mapping Theorem an invariance of L^1_2 spaces under a conformal homeomorphisms.

The Klein-Gordon Equation, the Hilbert Transform, and Dynamics of Gauss-type Maps

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This reports on joint work with A. Montes-Rodriguez.

We go beyond the earlier work (Ann. of Math. (2011)) and obtain the completeness the collection of powers of two atomic inner functions in the weak-star topology of H^∞. We note the ramifications for the Klein-Gordon equation in one spatial dimension, and the observed difference of space-like and time-like quarter-planes. In addition, we explore a new property, which we coin "dynamic unique continuation".

Convergence Almost Everywhere of Multiple Fourier Series

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We discuss convergence almost everywhere of Fourier series. We provide a new approach which allows us to prove the novel interpolation estimates for the Carleson maximal operators generated by the partial sums of the multiple Fourier series and all its conjugate series over cubes defined on the d-dimensional torus $\mathbb{T}^d$. Combing these estimates we show that these operators are bounded from a variant of the Arias-de-Reyna space QA^d to the weak L^1-space on $\mathbb{T}^d$. This implies that the multiple Fourier series of every function $f \in QA^d$ and all its conjugate series converge over cubes almost everywhere. By a close analysis of the space QA^d we prove that it contains a Lorentz space that strictly contains the Orlicz space $L(\log L)^d \log\log\log L(\mathbb{T}^d)$. As a consequence we obtain an improvement of a deep theorem proved by Antonov which was the best known result on the convergence of multiple Fourier series over cubes. The talk is based on the joint work with L. Rodríguez-Piazza.

Sobolev inequalities on arbitrary domains

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A theory of Sobolev inequalities in arbitrary open sets in $\mathbb{R}^n$ is offered. Boundary regularity of domains is replaced with information on boundary traces of trial functions and of their derivatives up to some explicit minimal order. The relevant Sobolev inequalities involve constants independent of the geometry of the domain, and exhibit the same critical exponents as in the classical inequalities on regular domains. Our approach relies upon new representation formulas for Sobolev functions, and on ensuring pointwise estimates which hold in any open set. This is a joint work with A. Cianchi.

Multilinear Integral Operators in Some Non-standard Weighted Function Spaces

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Our aim is to present the boundedness results regarding the one-weight theory of multi(sub) linear maximal, Calderón-Zygmund and potential operators in grand Lebesgue spaces. The spaces and operators are defined, generally speaking, on quasi-metric spaces with doubling measure. Two-weight weak and strong type criteria for multisublinear maximal type operators in Banach function lattices are also derived. The results are derived jointly with V. Kokilashvili and M. Mastyło.

Potential Estimates for Quasilinear Parabolic Equations

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Let us consider the following homogeneous quasilinear parabolic equations whose prototypes are the p-Laplacian ($\frac{2N}{N+1} < p < \infty$) and the Porous medium equation ($(\frac{N-2}{N})_+ < m < \infty$).

$$u_t = \operatorname{div} A(x,t,u,Du), \quad (x,t) \in \mathbb{R}^N \times [0,+\infty), \tag{1}$$

where the functions $A := (A_1, \ldots, A_N)$ are assumed to be only measurable in $(x,t) \in \mathbb{R}^N \times [0,+\infty)$, continuous with respect to u and Du for almost all (x,t).

By using recent results obtained in collaboration with Bögelein, Calahorrano, Piro Vernier and Ragnedda we are able to give sharp pointwise estimates from above and from below starting from the value of the solution attained in a point. These estimates generalise the classical estimates due to Moser. We apply these results to give sharp estimates to the fundamental solution of such class of equations

Short Communications

On the Well-Posed of General Boundary Value Problem for Non-linear Impulsive Systems with Fixed Impulses Points

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We present the results concerning to the well-posed question for the system of nonlinear impulsive equations with finite number of impulses points

$$\frac{dx}{dt} = f(t,x) \ \text{ almost everywhere on } \ [a,b] \setminus \{\tau_1, ..., \tau_{m_0}\}, \tag{2}$$

$$x(\tau_l+) - x(\tau_l-) = I_l(x(\tau_l)) \ \ (l = 1, \dots, m_0) \tag{3}$$

under the general boundary value condition

$$h(x) = 0, \tag{4}$$

where $a < \tau_1 < ... < \tau_{m_0} < b$, m_0 is a natural number, f is a vector-function from the Carathéodory class $Car([a,b] \times \mathbb{R}^n, \mathbb{R}^n)$, and $I_l : \mathbb{R}^n \to \mathbb{R}^n$ $(l = 1, ..., m_0)$ and $h : C([a,b], \mathbb{R}^n; \tau_1, ..., \tau_{m_0}) \to \mathbb{R}^n$ are continuous, nonlinear in general, operators.

The sufficient (among them the effective) conditions are presented guaranteing both the solvability of the impulsive boundary value problems

$$\frac{dx}{dt} = f_k(t,x) \ \text{ almost everywhere on } \ [a,b] \setminus \{\tau_{1k}, ..., \tau_{m_0 k}\},$$

$$x(\tau_{lk}+) - x(\tau_{lk}-) = I_{lk}(x(\tau_{lk})) \ \ (l = 1, ..., m_0);$$

$$h_k(x) = 0$$

$(k = 1, 2, ...)$ for any sufficient large k and the convergence of its solutions to a solution of the problem (1), (2);(3) as $k \to +\infty$, where $f_k \in Car([a,b] \times \mathbb{R}^n, \mathbb{R}^n)$, $a < \tau_{1k} < ... < \tau_{m_0 k} < b$, and I_{lk} $(l = 1, ..., m_0)$ and h_k are continuous operators.

The well-posed problem for the general linear boundary value problem for impulsive systems with finite number of impulses points is investigated in [1], where the necessary and sufficient conditions are given for the case. Analogous problems are investigated in [2], [3] (see also the references therein) for the linear and nonlinear boundary value problems for ordinary differential systems.

[1] M. Ashordia, G. Ekhvaia, Criteria of correctness of linear boundary value problems for systems of impulsive equations with finite and fixed points of impulses actions. *Mem. Differential Equations Math. Phys.* **37** (2006), 154-157.

[2] M. T. Ashordia, On the stability of solutions of linear boundary value problems for a system of ordinary differential equations. *Georgian Math. J.* **1** (1994), No.2, 115-126.

[3] I.T. Kiguradze, Boundary value problems for systems of ordinary differential equations. (English) *J. Sov. Math.* **43** (1988), No.2, 2259-2339; translation from Itogi Nauki Tekh., *Ser. Sovrem. Probl. Mat., Novejshie Dostizh.* **30**, (1987), 3–103.

Some Properties of Exotic Point Sets with Respect to Invariant Measures

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In this thesis our discussion is devoted to certain exotic point sets of the real line $\mathbf{R}$, in particular, to Vitali sets, Bernstein sets, Hamel bases, Luzin sets and Sierpinski sets and their properties with respect to invariant measures. The general measure extension problem is to extend a given measure μ onto a maximally large family of subsets of $\mathbf{R}$. It is known that:

(a) Vitali sets are not measurable with respect to any measure on $\mathbf{R}$ which extends the Lebesgue measure λ and is invariant under the group of all translations of $\mathbf{R}$.

(b) Any Hamel basis of $\mathbf{R}$ is absolutely negligible with respect to translation invariant (quasi-invariant) measures on $\mathbf{R}$.

(c) Luzin sets are very small from the point of view of measure theory since they have measure zero with respect to the completion of any σ-finite diffused Borel measure on $\mathbf{R}$.

Here we are dealing with the above-mentioned sets in light of function theory and measure theory, and establish some interrelations between them. In particular, it is shown that:

1. Some Bernstein set X is relatively measurable with respect to the class of all translation invariant measures on $\mathbf{R}$ extending λ, but X does not possess the uniqueness property with respect to the same class of measures;
2. There exists a translation invariant measure μ on $\mathbf{R}$ extending the standard Lebesgue measure and such that all Sierpinski sets are of μ-measure zero;
3. There exists some Bernstein set which is absolutely negligible with respect to the class of all σ-finite translation invariant (quasi-invariant) measures on $\mathbf{R}$.

References:

[1] Cichon J., Kharazishvili A., Werglorz B. *Subsets of the Real Line*, Wydawnictwo Uniwersytetu Lodzkiego, Lodz, 1995.

[2] J.C. Oxtoby, *Measure and Category*, Springer-Verlag, New York, 1971.

[3] M. Beriashvili, *On some paradoxical subsets of the real line*, Georgian International Journal of Science and Technology, Volume 6, Number 4, 2014, pp. 265275.

[4] A. B. Kharazishvili, *Nonmeasurable Sets and Functions*, Elsevier, Amsterdam, 2004.

Natural and Engineering nonuniform flows as dynamical systems: issues and challenges

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Nonuniform flows are ubiquitous both in nature and in the laboratory: they occur in atmospheres, oceans, stars, protoplanetary disks, galaxies, pipe flows, tokamak reactors and etc. The understanding of nonuniform kinematics associated with dynamical phenomena is as equally important as thermodynamic inhomogeneity. Indeed, often, the appearance of complex dynamics is a manifestation of nonuniform kinematics.

The non-normal nature of shear flows and its consequences became well understood by the hydrodynamic community in the 1990s. Shortcomings of traditional modal analysis (spectral expansion of perturbations in time and, later, eigenfunction analysis) for shear flows have been revealed. Operators in the mathematical formalism of shear flow modal analysis, such as for plane Couette and Poiseuille flows, are non-normal and the corresponding eigenmodes are nonorthogonal. The nonorthogonality leads to strong interference among the eigenmodes. Consequently, a proper approach should fully analyze eigenmode interference. While possible in principle, this is in practice a formidable task. The mathematical approach was therefore changed, and a breakthrough in the understanding and precise description of linear transient phenomena ensued. It was disclosed that the non-normality of the operators results in two novel *transient* channels of energy exchange in the linear theory of smooth (without inflection point) spectrally stable shear flows. Through the first channel, perturbations transiently exchange energy with a mean flow. The second channel-linear mode coupling ensures energy exchange among different modes/types of perturbations, for example, between vortices and waves.

The problem of the onset and self-sustenance of turbulence in spectrally stable shear flows is a challenge to fluid dynamics research. To the end of the last century it also became clear that, in spectrally stable shear flows the imperfect linear transient growth must be compensated by the nonlinear positive feedback that repopulates perturbations having potential of transient growth. As a result, the nonlinearity closes the feedback loop, producing self-sustaining perturbations. On the basis of the interplay between linear transient growth and nonlinear positive feedback, the hydrodynamic community formulated the bypass transition concept to explain the onset of turbulence in spectrally stable shear flows. Classical (direct and inverse) nonlinear cascade processes, even if anisotropic, are in fact unable to provide self-sustenance of perturbations (turbulence) when just transiently growing modes are present in the flow. However, in some shear flows, comes into play, so-called, nonlinear transverse cascade (NTC), that is, transverse/angular redistribution of perturbation harmonics in $\mathbf{k}$-space. In this case, turbulence can self-organize and be self-sustained as the flow shear continuously supply the turbulence with energy, thanks to an essential constructive feedback provided by the NTC.

Asymptotic Analysis of Dynamical Mixed Problems of Electro-Magneto-Elasticity in Domains with Cracks

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We investigate the solvability and asymptotic properties of solutions to 3- dimensional dynamical mixed initial boundary-value problems of electro-magneto-elasticity for homogeneous anisotropic bodies with cracks. Using the Laplace transform, potential theory and theory of pseudodifferential equations on a manifold with boundary, the existence and uniqueness theorems are proved. The complete asymptotics of solutions are obtained near the crack edges and near the lines where the different boundary conditions collide. We analyse singularity properties of solutions and the corresponding stresses. We have found an important special class of transversally-isotropic bodies for which the oscillating singularities vanish and the singularity exponents are calculated explicitly with the help of the eigenvalues of a special matrix associated with the principal homogeneous symbol matrix of the corresponding pseudodifferential operator. It turned out that these singularity exponents essentially depend on the material constants.

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Asymptotic Analysis of Fundamental Solutions of Hypoelliptic Equations

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Radiation conditions were first derived by A. Sommerfeld for Helmholtz operator [1] and subsequently were generalized in the following papers [2], [3]. Here are obtained Sommerfeld type conditions at infinity for polymetaharmonic equations, which ensure uniqueness of solutions in $\mathbb{R}^n$. In the paper [2] is studied uniqueness of solution of the polymetaharmonic equation, where characteristic polynomial has multiple zeros.
In the monograph [4] were obtained radiation conditions for hypoelliptic differential equations, where characteristic polynomials have real simple zeros.

We generalize the results obtained in [4] and consider the case when the corresponding characteristic polynomials of the hypoelliptic differential equations have real multiple zeros. We investigate asymptotic properties at infinity of fundamental solutions of the hypoelliptic differential equations. On the basis of asymptotic analysis of fundamental solution we find conditions at infinity, which ensure that these equations are uniquely solvable.

References:

[1] A.Sommerfeld, *Partial differential equations in physics*, Acad. Press N.Y. 1949.

[2] B.Paneah, *On existence and uniqueness of the solution of n-metaharmonic equation in unbounded space,* Mosc. State Univ. Vestnic, Math. Mech. No5,(1959), 123-135.

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[4] B.R. Vainberg, *Asymptotic methods in equations of mathematical physics*, Gordon and Breach Science Publishers, New York London Paris Montreux Tokyo Melbourne, 1989.

On Some Partial Differential and Integro-Differential Nonlinear Models

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One very important partial differential and integro-differential nonlinear model is obtained at mathematical simulation of processes of electro-magnetic field penetration in the substance. In the quasi-stationary approximation, the corresponding system of Maxwell's equations has the form:

$$\frac{\partial H}{\partial t} = -rot\left(\nu_m rot\, H\right), \tag{5}$$

$$c_\nu \frac{\partial \theta}{\partial t} = \nu_m \left(rot\, H\right)^2, \tag{6}$$

where $H = (H_1, H_2, H_3)$ is a vector of magnetic field, θ – temperature, c_ν and ν_m characterize correspondingly heat capacity and electroconductivity of the medium. System (1) describes propagation of magnetic field in the medium, and equation (2) describes temperature change at the expense of Joule's heating without taking into account of heat conductivity.

Maxwell model (1), (2) is complex for investigation and practical study of certain diffusion problems, therefore its comparatively simplified versions are often used and studied.

If c_ν and $\nu_m$depend on temperature θ, i.e. $c_\nu = c_\nu(\theta)$, $\nu_m = \nu_m(\theta)$, then the system (1), (2) as it was done in the work - Gordeziani D.G., Dzhangveladze T.A., Korshia T.K. Existence and Uniqueness of a Solution of Certain Nonlinear Parabolic Problems. Differential'nye Uravnenyia, 1983, V.19, N7, p.1197-1207 - can be rewritten in the following form:

$$\frac{\partial H}{\partial t} = -rot\left[a\left(\int_0^t \left|rot\, H\right|^2 d\tau\right) rot\, H\right], \tag{7}$$

where coefficient $a = a(S)$ is defined for $S \in [0, \infty)$.

Many scientific works were dedicated to the investigation of (3) type integro-differential models.

For more thorough description of electromagnetic field propagation in the medium, it is desirable to take into consideration different physical effects, first of all - heat conductivity of the medium. In this case, again with taking into account of Joule law, instead of equation (2), the following equation is considered

$$c_\nu \frac{\partial \theta}{\partial t} = \nu_m \left(rot\, H\right)^2 + \nabla\left(k \nabla \theta\right), \tag{8}$$

where k is heat conductivity coefficient, which also depends on temperature. Some aspects of investigation and numerical resolution of one-dimensional version of system (1), (4), in case of one-components magnetic field, are given for example in the work - Abuladze I.O., Gordeziani D.G., Dzhangveladze T.A., Korshia T.K. Discrete Models for a Nonlinear Magnetic-field Scattering Problem with Thermal Conductivity. Differential'nye Uravnenyia, 1986, V.22, N7, p.1119-1129.

Many scientific papers are devoted to the construction and investigation of discrete analogues of above-mentioned differential and integro-differential models. There are still many open questions in this direction.

We study some properties of solutions of initial-boundary value problems for one-dimensional analog of systems (3) and (4) as well as numerical approximation of those problems for special nonlinearities.

The Riemann and Green-Hadamard Functions of Linear Hyperbolic Equations and Their Applications

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Some new properties of the Riemann and Green-Hadamard functions of linear second order hyperbolic equations of general form and their applications are considered

The Local and Global Solvability of a Multidimensional Boundary Value Problem for Some Second Order Semilinear Hyperbolic Systems

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For one class of the second order semilinear hyperbolic systems it is investigated a multidimensional boundary value problem in a light cone of the future. The questions of local and global solvability of this problem are considered.

On Non-separable Extensions of Dynamical Systems

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As is known, an abstract dynamical system can be treated as a triplet (E, G, μ), where E is a nonempty phase space, G is some group of transformations of E, and μ is a nonzero finite (or σ-finite) G-invariant measure defined on some G-invariant σ-algebra of subsets of E.

One of the important topics in contemporary abstract dynamical system theory (or,equivalently, in the theory of invariant measures) is concerned with the general problem of the existence of a nontrivial continuous dynamical system on a sufficiently large class of subsets of an initial base set E which is usually assumed to be uncountable. In general, it is impossible to define a nonzero continuous dynamical system on the family of all subsets of E. It follows from this observation that, for any such dynamical system, the class of sets $\mathrm{dom}(\mu)$ is relatively poor. So the natural question arises whether it is possible to essentially expand the above-mentioned class of sets. Various methods are known of expanding an original dynamical system (E, G, μ) by constructing G-invariant extensions of μ. In this way, one can get even non-separable extensions of given dynamical systems see, e. g., [1],[2]).

Let (E_1, G_1, μ_1) and (E_2, G_2, μ_2) be two dynamical systems. We recall that a graph $\Gamma \subset E_1 \times E_2$ is $(\mu_1 \otimes \mu_2)$-thick in $E_1 \times E_2$ if, for each $(\mu_1 \otimes \mu_2)$-measurable set $Z \subset E_1 \times E_2$ with $(\mu_1 \otimes \mu_2)(Z) > 0$, we have $\Gamma \cap Z \neq \emptyset$.

Theorem. *Let (E_1, G_1, μ_1) and (E_2, G_2, μ_2) be two dynamical systems and let there exists a mapping $f : E_1 \to E_2$ satisfying the following conditions:*

[1] *for any element $g_1 \in G_1$ there exists element $g_2 \in G_2$ such that $f \circ g_1 = g_2 \circ f$;*

[2] *the graph of f is a $(\mu_1 \otimes \mu_2)$-thick subset of the $E_1 \times E_2$.*

[3] *μ_2 is a probability measure.*

Then there exists a dynamical system (E, G, μ) such that:

(a) *(E, G, μ) extends the initial dynamical system (E_1, G_1, μ_1);*

(b) *if (E_2, G_2, μ_2) is non-separable, then (E, G, μ) is non-separable too.*

References:

[1] A. B. Kharazishvili, *Topics in Measure Theory and Real Analysis,* Atlantic Press, Amsterdam-Paris, 2009.

[2] M. Beriashvili, A. Kirtadze, *On the uniqueness property of non-separable extensions of invariant Borel measures and relative measurability of real-valued functions,* Georgian Math. Journal, vol.29, no.1, 2014, pp. 49-57.

Sharp Direct and Inverse Theorems on Approximation of Functions in Weighted Variable Exponent Lebesgue Spaces on the Real Line

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In our talk we present some extensions of fundamental inequalities (Bernstein-Zygmund, Nikol'skii) for entire functions of finite order in weighted variable exponent Lebesgue spaces on the real line. On the base of these inequalities two-sided sharp estimates for generalized moduli of smoothness in terms of the best approximations by the entire functions of finite order are established. The generalized moduli of smoothness is defined via the Steklov means. In particular, the obtained estimates state space metric influence on the rate of decrease to zero of smoothness moduli.

Some Geometrical Properties of Variable Exponent Lebesgue Spaces

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In the present talk we will discuss some geometrical properties of the norms in Banach function spaces. Particularly, if exponent $1/p(\cdot)$ belongs to $BLO^{1/\log}$ then for the norm of corresponding variable exponent Lebesgue space we have the following lower estimate

$$\left\| \sum \frac{\|f\chi_Q\|_{p(\cdot)}}{\|\chi_Q\|_{p(\cdot)}} \chi_Q \right\|_{p(\cdot)} \leq C\|f\|_{p(\cdot)}$$

where $\{Q\}$ defines any disjoint partition by intervals of $[0;1]$ and C is an absolute constant not depended on the partition and function. Also we have constructed variable exponent Lebesgue space with above property which does not possess following upper estimation

$$\|f\|_{p(\cdot)} \leq C \left\| \sum \frac{\|f\chi_Q\|_{p(\cdot)}}{\|\chi_Q\|_{p(\cdot)}} \chi_Q \right\|_{p(\cdot)}.$$

Let $\mathcal{B}$ defines the set of all exponents for which Hardy-Littlewood maximal operator is bounded in space $L^{p(\cdot)}[0;1]$. It is well known that if $p(\cdot) \in \mathcal{B}$ then $p(\cdot) \in BMO^{1/\log}$. On the other hand if $p(\cdot) \in BMO^{1/\log}$, $(1 < p_- \leq p_+ < \infty)$, then there exists $c > 0$ such that $p(\cdot) + c \in \mathcal{B}$. Also There exists exponent $p(\cdot) \in BMO^{1/\log}$, $(1 < p_- \leq p_+ < \infty)$ such that $p(\cdot) \notin \mathcal{B}$. From this follows that in general we can not assume that $c = 0$.

The same question is interesting in case of $1/p(\cdot) \in BLO^{1/\log}$, namely:

Question A. Let for exponent $p(\cdot)$, $1 < p_- \leq p_+ < \infty$ and for $c > 0$ we have $1/(p(\cdot) + c) \in BLO^{1/\log}$ and $p(\cdot) + c \in \mathcal{B}$. Does this imply $p(\cdot) \in \mathcal{B}$?

Question B. Let for exponent $p(\cdot)$, $1 < p_- \leq p_+ < \infty$ and for $c > 1$ we have $1/(cp(\cdot)) \in BLO^{1/\log}$ and $cp(\cdot) \in \mathcal{B}$. Does this imply $p(\cdot) \in \mathcal{B}$?

We construct exponent $p(\cdot)$, $(1 < p_- \leq p_+ < \infty)$, $1/p(\cdot) \in BLO^{1/\log}$ such that the Hardy-Littlewood maximal operator is not bounded in $L^{p(\cdot)}[0;1]$. This gives the negative answer on the above questions.

Also there has been shown that for constructed exponent the classical Hardy operator

$$Tf(x) = \frac{1}{x} \int_0^x f(t)dt$$

is not bounded from $L^{p(\cdot)}[0;1]$ to $L^{p(\cdot)}[0;1]$.

Characterization of Linear Differential Systems (in Several Variables)

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It was demonstrated by several authors that a discrete-time dynamical system can be represented by a linear constant coefficient partial difference equation if and only if it is linear, shift-invariant and complete. In this talk, this result will be extended to the continuous time case. It turns out that the same properties (linearity, shift-invariance and completeness) characterize as well continuous-time dynamical systems, by which one means linear differential systems (i.e., the solution sets of linear constant coefficient partial differential equations). The continuous-time version of the completeness property is a bit more complicated due to the presence of flat functions in the space of smooth functions, and takes into account jet-spaces at all points, but not at one point only. Using pure algebra, it will be shown then that a linear differential system can be completely reconstructed from knowledge of its k-jets at one fixed point, where k is a sufficiently large integer.

Rotation of Coordinate Axes and Differentiation of Integrals with Respect to Translation Invariant Bases

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The dependence of differentiation properties of an indefinite integral on a rotation of coordinate axes is studied, namely: the result of J. Marstrand on the existence of a function the integral of which is not strongly differentiable for any choice of axes is extended to Busemann-Feller and homothecy invariant bases which does not differentiate $L(\mathbb{R}^n)$; it is proved that for an arbitrary translation invariant basis B formed of multi-dimensional intervals and which does not differentiate $L(\mathbb{R}^n)$, the class of all functions the integrals of which differentiate B is not invariant with respect rotations, and for bases of such type it is studied the problem on characterization of singularities that may have an integral of a fixed function for various choices of coordinate axes.

The Riemann-Hilbert Problem for Generalized Analytic Functions in the Smirnov Class with Variable Exponent

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Let Γ be piecewise smooth simple close curve bounded the finite domain D and $A(z)$, $B(z)$ are given function from $L^{S,2}(D)$, $S>2$.

The Riemann-Hilbert problem

$$\Re[\lambda(t)W^{+}(t)] = b(t)$$

is considered in the Smirnov class $E^{p(t)}(A;B;D)$ where $E^{p(t)}(A;B;D)$ is the set of such regular solutions of the equation

$$\partial_{\overline{z}}W(z) + A(z)W(z) + B(z)\overline{W(z)} = 0$$

for which $W \in U^{S,2}(A;B;D)$, $S>2$ and

$$\sup_{0\le r<1} \int_0^{2\pi} |W(z(re^{i\alpha}))|^{P(z(e^{i\alpha}))} |z'(re^{i\alpha})d\alpha| < \infty$$

Here $z = z(re^{i\alpha})$ is function conformal mapping the circle $U = \{w : \ |w|<1\}$ on D.

Supposed that $a(t) = \lambda(t)/\overline{\lambda(t)}$ belongs to the class $\Lambda(\rho(t),\Gamma)$ which

On Dynamical Systems in a Polish Group

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Definition 1. Let $(G, \rho, \odot)$ be a Polish group, by which we mean a group with a complete separable metric ρ for which the transformation (from $G \times G$ onto G) which sends (x, y) into $x^{-1} \odot y$ is continuous. Let $B(G, \rho)$ denotes the σ-algebra of Borel subsets of G defined by the metric ρ.

Definition 2. A triplet (G, Γ, μ) is called dynamical system in a Polish group $(G, \rho, \odot)$, if Γ is a group of Borel measurable transformations of G and μ is Γ-invariant σ-finite non-zero Borel measure in G.

If $\Gamma = \{\gamma_h : G \to G | h \in G \ \& \ (\forall g)(g \in G \to \gamma_h(g) = h \odot g)\}$, then μ is called a left Haar measure in G.

Definition 3. We say that a diffused Borel probability measure μ defined in a Polish space (G, ρ)(for which a group operation is not defined) is a left Haar measure if there exist a metric ρ_1 and a group operation $\odot$ in G such that $(G, \rho_1, \odot)$ stands a compact Polish group with a left Haar measure μ for which $B(G, \rho) = B(G, \rho_1)$.

By using Borel isomorphism theorem of two probability diffused Borel measures with domain in Polish spaces (see [1], Theorem 4.12, p. 81) we prove the following statement.

Theorem 1. Let μ be a diffused Borel probability measure defined on a Polish space (G, ρ). Then there exists a metric ρ_1 and a group operation $\odot$ in G such that $(G, \rho_1, \odot)$ stands a compact Polish group with a left Haar measure μ for which $B(G, \rho) = B(G, \rho_1)$.

Remark 1. There naturally arises a question asking whether the metric ρ_1 and a group operation $\odot$ in G participate in the formulation of Theorem 1 are unique. We show that the answer to this question is *no*.

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Operators with Rough Kernels in Variable Exponent Spaces

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In this talk, we study boundedness results of some rough operators in the framework of variable exponent Lebesgue and Morrey spaces. It is shown that the maximal operator, fractional maximal operator, sharp maximal operators and fractional operators are bounded operators.

The talk is based on joint work with Stefan Samko.

The Wavelet Characterization of Variable Exponent Lebesgue Space

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Denote by $\mathcal{B}^{loc}(\mathbb{R}^n)$ the class of all measurable functions $p : \mathbb{R}^n \longrightarrow [1, \infty)$ for which local variant of Hardy-Litlewood maximal operator M^{loc} is bounded on $L^{p(\cdot)}(\mathbb{R}^n)$.

In this talk we will discuss the wavelet characterization of variable exponent Lebesgue space in case of $p(\cdot) \in \mathcal{B}^{loc}(\mathbb{R})$. Inhomogeneous wavelets of Daubechies type are considered. Some conditions for exponents are found for which the wavelet system is an unconditional basis in $L^{p(\cdot)}(\mathbb{R}^n)$ space.

On Weighted Boundedness of the Maximal Operator in Local Morrey Spaces

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We consider the problem of weighted boundedness of the maximal operator in Morrey spaces. As is known, the inclusion of a weight into the Muckenhoupt class A_p is neither sufficient nor necessary for such a boundedness. For two weighted local Morrey spaces $\mathcal{L}^{p,\varphi}_{\{x_0\}}(\Omega, u)$ and $\mathcal{L}^{p,\varphi}_{\{x_0\}}(\Omega, v)$ we obtain general type sufficient conditions and necessary conditions imposed on the functions φ and ψ and the weights u and v for the boundedness of the maximal operator from $\mathcal{L}^{p,\varphi}_{\{x_0\}}(\Omega, u)$ to $\mathcal{L}^{p,\varphi}_{\{x_0\}}(\Omega, v)$, with some "logarithmic gap" between the sufficient and necessary conditions. Both the conditions formally coincide if we omit a certain logarithmic factor in these conditions.

Variable Exponent Sobolev Theorem for Fractional Integrals on Quasimetric Measure Spaces

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We show that the fractional operator $I^{\alpha(\cdot)}$ of variable order on a bounded open set Ω in a quasimetric measure space (X, d, μ) with the growth condition the measure μ, is bounded from the variable exponent Lebesgue space $L^{p(\cdot)}(\Omega)$ into $L^{q(\cdot)}(\Omega)$ in the case $\inf_{x\in\Omega}[n(x) - \alpha(x)p(x)] > 0$, where $\frac{1}{q(x)} = \frac{1}{p(x)} - \frac{\alpha(x)}{n(x)}$ and $n(x)$ comes from the growth condition under the log-continuity condition on $p(x)$.

The BMO-result is also obtained in the case $\alpha(x)p(x) \equiv n(x)$.

Identification and Modelling of One Class of Dynamical Systems

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Structural and parametric identification of the nonlinear continuous dynamic systems with a closed cycle on the set of continuous block-oriented models with feedback is considered. Systems, usually functioning with a closed cycle are widespread in chemical, mining, pulp-and-paper industry, ecologies, etc. They are complex nonlinear control objects - the steady movement at their output is reached only at the certain values of the parameters of the system and under the change of the input influence within certain limits. Problem of structural identification, which is coordinated with L. Zadeh's classical definition of identification, is posed as follows: classes of models and input signals are given; it is required to develop a criterion identifying the model structures from the set of models. Thus the a priori information is applied to the task of a class of models, and a posteriori information - for definition of structure of model from this set. Models of systems with closed cycle, taking into account the peculiarities of their operation, are described by the ordinary nonlinear differential equations - Riccati and Duffing equations. Input signals are periodic functions, having absolutely and uniformly convergence trigonometrical series. The method of the structural identification in the steady state based on the observation of the system's input and output variables at the input periodic influences is proposed. According to the system stability conditions and Diulak criterion, for the solution of Riccati and Duffig equations, corresponding to the models, it is possible to use the method of a small parameter. On the basis of application of the numerical harmonic analysis the definition criterion of model structure on the set of models is developed. The solution of the parameter identification problems, which can be immediately connected with the structural identification problem, is carried out in the steady and transient states by the method of least squares. The structural and parametric identification algorithms are investigated by means of both the theoretical analysis and the computer modelling.

Almost Periodic Factorization: A Survey

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Factorization of matrix functions (that is, their representation as products of multiples analytic inside and outside the given closed curve) arises naturally in many applications, including those to convolution type equations on a half-line (the classical Wiener-Hopf method). As it happens, the equations on finite intervals also can be treated via the factorization method. The resulting matrix functions, however, are of oscillating type, which has not been treated until much later. The general case can be boiled down to the situation when the matrix is almost periodic, that is, its elements belong to the algebra generated by $\exp(iax)$ with real values of the parameter a. We will discuss the current state of the factorization problem for such matrices, using in particular [1–7]. A special attention will be paid to a (seemingly narrow) case of 2-by-2 triangular matrix functions, but even for them the factorability properties remain a mystery in striking difference with both the scalar almost periodic case and the classical Wiener-Hopf setting.

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Sensitivity Analysis for Some Classes of Controlled Dynamical Systems with Time-Delay

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Sensitivity analysis consists in the investigation of initial data perturbations influence on a solution of differential equation. For the time-delay controlled functional differential equation

$$\dot{x}(t) = f(t, x(t), x(t - \tau_0), u_0(t)), t \in [t_{00}, t_1]$$

with the discontinuous initial condition

$$x(t) = \varphi_0(t), t \in [t_{00} - \tau_0, t_{00}), x(t_0) = x_{00}$$

linear representation of the first order sensitivity coefficient is obtained with respect to perturbations of initial data $(t_{00}, \tau_0, x_{00}, \varphi_0(t), u_0(t))$. Analogous representations for various classes of functional differential equations are proved in [1,2]. Moreover, for the initial data optimization problems the necessary conditions of optimality are obtained.
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On Uniqueness of Functional Series

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It is well known that many deep researches are devoted to the uniqueness problem of functional series with respect to different systems of functions. According to Cantor's fundamental result the empty set is a set of uniqueness (a U set) for trigonometric series. This theorem was generalized by Young, who proved that any countable set is a U set for trigonometric series.

In the presented paper we formulate some of our theorems connected with some properties of uniqueness sets of functional series.

Let $\Phi = \{\varphi_n(x)\}_{n=1}^{\infty}$ be a system of finite functions defined on $[0,1]$. By m we denote the set of all number sequences and by a and b we denote some elements of the set m. So,

$$a = (a_1, a_2, \ldots, a_n, \ldots) \quad \text{and} \quad b = (b_1, b_2, \ldots, b_n, \ldots).$$

We say that $a = b$ if and only if $a_n = b_n$ for every whole number $n \geq 1$. The sequence $(0, 0, 0, \ldots)$ we denote by θ. Let $m_0 = \{a \in m : \ a \neq \theta\}$. Consider a series with respect to Φ system

$$\sum_{n=1}^{\infty} a_n \varphi_n(x). \tag{9}$$

Definition 1. A set $A \subset [0,1]$ is called a set of uniqueness, or a U set, if the equality

$$\sum_{n=1}^{\infty} a_n \varphi_n(x) = 0 \quad \text{for all} \quad x \in [0,1]/A$$

implies that $a_n = 0$ for every whole $n \geq 1$. Otherwise the set A is called an M set.

Definition 2. We say that a set $E \subset [0,1]$ belongs to a class $V(\Phi)$ if the equality

$$\sum_{n=1}^{\infty} a_n \varphi_n(x) = 0 \quad \text{for all} \quad x \in E$$

implies that $a_n = 0$ for every whole number $n \geq 1$.

It is obvious that if $E \in V(\Phi)$, then the set $[0,1] \backslash E$ is a U set and if $E \notin V(\Phi)$, then the set $[0,1] \backslash E$ is an M set. Below everywhere $V(\Phi) \neq \emptyset$.

Let formulate some of our theorems.

Theorem 1. A set $E \subset [0,1]$ belongs to $V(\Phi)$ if and only if for any series (1), where $a \in m_0$, there exists a point $x^*(a) \in E$, such that

$$\sum_{n=1}^{\infty} a_n \varphi_n(x^*(a)) \neq 0.$$

Theorem 2. A set A is a U set if and only if

$$A \subset [0,1] \backslash \bigcup_{a \in m_0} \{x^*(a)\}.$$

A Note on the Structure of Atomic Components of Independent Families of Sets

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It is well known that independent families of sets play an important role in different branches of contemporary mathematics, especially, in general topology and mathematical analysis (see, for instance, [1], [2] and [3]). In [4] and [5] we investigated independent families of sets not only from set-theoretical view-point, but also from geometrical one. According to one of our results, there exists an uncountable independent family of convex compact figures in the Euclidean plane (see, [4] and [5]). Studies in this direction show that it is natural to answer the following question: how the structure of atomic components of any independent family of convex compact figures in the Euclidean plane depends on the cardinality of the family? We established the following two results. **Theorem.** *If five or more convex compact figures in the Euclidean plane form an independent family of sets, then for any point A of any atomic component W of the family there exists a triangle ABC with positive area, such that the following inclusion holds:*

$$\triangle ABC \subset W.$$

Theorem. *There exist independent families F_1 and F_2 of convex compact figures in the Euclidean plane, such that card(F_1) = card$(F_2) = 4$ and one of the atomic components of the family F_1 is a singleton while one of the atomic components of the family F_2 is a line segment with positive length.*

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Generalized Grand Lebesgue Spaces

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We denote by $L_a^{p),\theta}(\Omega)$ the generalized grand Lebesgue space on set $\Omega \subseteq \mathbb{R}^n$:

$$L_a^{p),\theta}(\Omega) := \left\{ f : \sup_{0<\varepsilon<p-1} \left(\varepsilon^\theta \int_\Omega |f(x)|^{p-\varepsilon} [a(x)]^\varepsilon \, dx \right)^{\frac{1}{p-\varepsilon}} < \infty \right\}, \; p > 1, \; \theta > 0,$$

where a there is some weighting function. It is shown that the $L_a^{p),\theta}(\Omega)$ is an extension of the classical Lebesgue space $L^p(\Omega)$ if and only if $a \in L^p(\Omega)$.

Theorem. *Let $0 < \alpha < n$, $1 < p < \frac{n}{\alpha}$, $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$, $\theta > 0$ and a be a weight from $L^p(\mathbb{R}^n)$. The Riesz potential operator*

$$I^\alpha f = \int_{\mathbb{R}^n} \frac{\varphi(t)}{|x-t|^{n-\alpha}} \, dt$$

is bounded from $L_a^{p),\theta}(\mathbb{R}^n)$ to $L_{a^{p/q}}^{q),\, q\theta/p}(\mathbb{R}^n)$ if and only if there exists $\delta \in (0, \frac{p}{q'})$ such that

$$a^\delta \in A_{\frac{p-\delta}{p}\left(1+\frac{q}{p'}\right)}.$$

The talk is based on joint work with Stefan Samko.